John P. Burgess
Philosophical Logic.
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http://www.princeton.edu/~jburgess/PHILOGErrata.pdf.

This concise book is about some non-classical systems of (mainly sentential) logic and their philosophical bases. Chapter 1 summarizes classical logic. Chapter 2, 'Temporal Logic', begins by showing how temporal discourse can be represented in 'the *regimented* language' (14) of classical quantificational logic with quantification over times and a two-place predicate '<' symbolizing 'is earlier than'. Model theory for this language is outlined (14-15). Next, 'the *autonomous* approach', which employs tense operators, is discussed. The axioms, rules and theorems of a '*minimal* temporal logic' (21) fit to characterize 'any conceivable time order' (26) are set out. The discussion turns to 'some significant steps towards' specifying a logic 'correct…for classical physics' (26-7). Restrictions on temporal frames, their corresponding axioms and some resultant theorems are covered (26-32). Discussion of quantified temporal logic includes coverage of the tense-logical analogues of the Barcan and Converse Barcan Formulas (33-8).

Chapter 3, 'Modal Logic', covers the parallels between modal and temporal logics, Kripke models, and the sentential modal logics **K**, **T**, **S4**, **S4.2**, **S4.3** and **S5**. Those with a basic background in classical logic but not already familiar with modal logic would probably find it difficult to use this chapter for independent study. Nevertheless, the proofs provided of metalogical results for modal logics are short, elegant and clear. Discussion of quantified modal logic (67-70) concentrates on the Barcan and Converse Barcan Formulas and Quine's critique of quantified modal logic.

In Chapter 4, 'Conditional Logic', after some remarks on the view that indicative conditionals are material conditionals and on its Gricean critique (72-5), the probabilistic theory of indicative conditions is discussed (75-81). According to a criterion endorsed by Ernest Adams, an indicative conditional is assertible iff the probability of the consequent, given the antecedent, is high (77). In view of 'the *Lewis trivialization theorem*…it seems that the indicative conditional is *not* a compound the probability of whose truth is equal to the conditional probability of its consequent given its antecedent' (77-8). This gives rise to three main rival theories about the truth-conditions for indicative conditionals: (a) *materialism*, which says that the truth conditions of an indicative conditional match those of the corresponding material conditional; (b) *idealism*, which does not distinguish truth from assertibility; and (c) *nihilism* which 'maintains that indicative conditionals…do not have truth values but merely assertibility conditions' (78). Burgess (81-4) proceeds to 'the remoteness theory of indicative conditionals' which presents the probabilistic account 'in qualitative, model-theoretic form, based on the notion of degree of remoteness of epistemic possibilities from credibility' (94). The remoteness theory outlined is an adaptation of the Lewis/Stalnaker account of counterfactual conditionals to the case of indicative conditionals. Accounts of counterfactuals themselves are also covered (94-6).

Chapter 5, 'Relevantistic Logic', concerns logics which reject *ex falso quodlibet*. This requires giving up at least one of (i) disjunction introduction; (ii) disjunctive syllogism; (iii) the thesis that entailment is transitive (99-100). Whereas the expressions 'relevance logic' and 'relevant logic' belong 'to those who reject disjunctive syllogism', Burgess uses 'relevantistic' as a label for any logic that takes one of these three paths. As background, '*topic* logic' is introduced (100):

> In topic logic there are three additional two-place connectives _ and \ and /, where $A \_B$ and $A \setminus B$ and A / B are intended to symbolize that the subject matter of A overlaps [with], or is contained in, or contains the subject matter of B.

In classical logic, a conjunction of premises, A, entails a conclusion, B, iff $A \supset B$ is valid. Three alternative right-hand sides are discussed, each furnishing a relevantistic conception of entailment. 'Relatedness logic' requires that $(A \supset B) \wedge (A \_ B)$ be valid. The 'analytic implication' view of entailment requires that $(A \supset B) \wedge (A / B)$ be valid. The 'co-analytic implication' view requires that $(A \supset B) \wedge (A \setminus B)$ be valid.

As follows, each of these three non-classical conceptions of entailment embodies one of the three ways of rejecting *ex falso quodlibet* mentioned above: the co-analytic implication view (i); the analytic implication view (ii); relatedness logic (iii). These conceptions take entailment to require a certain kind of overlap. However, according to the more mainstream relevantists,

> the trio of sects under discussion err…. Each either counts $A \wedge \neg A \wedge B$ as entailing $\neg B$ or counts A as entailing $\neg A \vee B \vee \neg B$ or both, simply because the argument from premise conclusion is truth-preserving and the topic of the conclusion is appropriately related to the topic of the premises…. In neither case does the relation of topics have anything to do with *why* the argument is truth-preserving (102).

Mainstream relevantists require 'not only…an overlap of topic', but that 'the overlap…is what *makes* the argument truth-preserving' (ibid.).

Burgess rejects the co-analytic implication view because any objection to disjunction introduction on Gricean grounds can be countered by Gricean means (102-3) and because disjunction introduction is indispensible to mathematical practice (103).

Burgess discusses 'perfectionist logic' (103-7). A perfectly entails B iff A classically entails B, A is satisfiable and B is invalid (103).

> Perfection cannot be advocated as the criterion of entailment because there are imperfect arguments that are instances of perfect ones (such as the argument from p to $p \vee \neg p$ or from $q \wedge \neg q$ to q), and any instance of a valid form of argument must be recognized as a valid form of argument (ibid.).

The idea of perfection can be invoked without that drawback: 'A *perfectibly* entails B iff the argument from A to B is either perfect itself or a substitution instance of a perfect argument….*perfectionist* logic advocates perfectibility as the criterion of entailment' (ibid.).

Perfectionist logic accepts disjunction introduction and disjunctive syllogism, but employs a non-transitive notion of entailment (104): $p \wedge \neg p$ perfectibly entails $(p \vee q) \wedge \neg p$; $(p \vee q) \wedge \neg p$ perfectibly entails q; it is not the case that $p \wedge \neg p$ perfectibly entails q. A sequent is valid in perfectionist logic iff 'it is classically valid and no *proper subsequent*, obtained by dropping one or more formulas from one or the other or both sides [is] classically valid' (105).

In perfectionist sequent calculus, cut is not valid. Cut 'directly expresses the classical doctrine that entailment is transitive' (106). However, Gentzen's cut-elimination metatheorem shows that cut 'is in principle dispensable in sequent calculus' (ibid.). Any classical-logic proof of a mathematical theorem that uses cut can be transformed into a classical-logic proof that does not. From the latter proof, it is possible 'to extract…a perfectionistically acceptable sequent demonstration, if not that the original axioms entail the theorem, then either that a subset of the original axioms entail the theorem, or that the axioms entail a contradiction' (106-7). The trouble is, however, that Gentzen's metatheorem itself 'has not been, and it is doubtful if it ever will be' proved in a manner that does not require the transitivity of entailment (107).

Burgess proceeds to discuss 'entailments among truth-functional compounds' (107) in the more mainstream relevantist logics ('r-logics'): i.e., those that are usually called 'relevance logics' or 'relevant logics', which reject disjunctive syllogism (107-12). This is the '"first-degree" fragment' of r-logic (107). A classically valid sequent of truth-functional logic is expressible as a sequent with a formula in conjunctive normal form as premise and one in disjunctive normal form as conclusion, thus (ibid.):

$$(9)\ (\neg)p_1 \wedge \ldots \wedge \ldots (\neg)p_m \vdash (\neg)q_1 \vee \ldots \vee \ldots (\neg)q_n$$

To assess whether the premise r-entails the conclusion,

> [i]ntroduce sentence auxiliary sentence letters, so that each ordinary sentence letter p, q, r, … has an auxiliary sentence letter p', q', r', … as a *mate*. Replace each p_i or q_j that appears negated in (9) by its mate to obtain (9′). Then (9) holds in r-logic iff (9′) holds in classical logic (ibid.).

The r-invalidity of disjunctive syllogism is evident: 'the argument from $(p \vee q) \wedge \neg p$ to q becomes the classically invalid argument from $(p \vee q) \wedge \neg p'$ to q' (108).

After short discussions of applied r-logic (110-12) and dialethism (112-13), the chapter concludes with a discussion of the 'purely implicational' fragment of r-logic and the problems that arise on attempting to combine it with the 'first degree' fragment (114-19).

In Chapter 6, 'Intuitionistic Logic', Burgess explains Gödel's interpretation of the intuitionistic sentential logic **I** as a modal logic in which the box is interpreted as a provability operator and shows, via Kripke models, that a formula is a theorem of the intuitionistic logic **I** iff that formula's modal transformation is a theorem of **S4** (130-32). Soundness and completeness are demonstrated for **I** (131-5). Two sentential logics 'intermediate between **I** and classical sentential logic' are briefly covered, including the restrictions they place upon frames (135-6). Quantified intuitionistic logic is discussed (136-41).

This book is interesting and useful. It enables readers to learn much in a short time, is good value and has a pleasant physical design (though my copy did not have any lettering on the spine). It could be valuable for philosophers working in metaphysics and the philosophy of language who are not logic specialists.

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