Matthew E. Moore, ed.
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Charles Sanders Peirce (1839–1914) was best known for his theory of pragmatism and for his semiotic theory (theory of signs). He was also a first-rate mathematical logician; his philosophy of mathematics builds on his intricate knowledge of mathematics and his own robust metaphysical and epistemological systems. He simultaneously believed that mathematics was a science much like any other, though with a unique subject matter, and that mathematics was at the heart of all knowledge. As evidence of the latter view, he saw metaphysical continuity as the fundamental basis for all knowledge, and spent much of his life trying to refine and defend a precise mathematical definition of continuity.

New Essays on Peirce's Mathematical Philosophy, edited by Matthew E. Moore, is a collection of eleven new essays on Peirce's philosophy of mathematics, and is a timely and thorough introduction to and analysis of Peirce's mathematical thought. To my knowledge it is the first such collection of papers on Peirce's mathematical philosophy, and such a treatment is long overdue. Moore correctly writes in the introduction that 'the mathematical dimensions of Peirce's philosophy…have been *unduly* neglected' (1). That Peirce is an important philosopher, and that mathematics was central to his philosophy, should give us reason enough for this volume. In addition, however, Peirce's mathematical ideas were quite philosophically innovative and unique, and deserve careful study. The articles of this volume should be of interest to those who study philosophy of mathematics, and Peircean scholars, as well as the intersection.

The articles can be divided roughly into three different subject areas. Taking them sequentially as they appear in the volume, the first set of articles (Hookway, Shin, Pietarinen, Tiercelin) attempts to place Peirce's philosophy of mathematics in historical context, largely by comparing Peirce's theories to modern schools of thought. The second set of essays (Campos, Marietti, Cooke) focuses on Peirce's unique analysis of mathematical experience and creation, each essay focusing on a different aspect of mathematical theory-making. A third collection of essays (Zalamea, Ehrlich, Havenel) analyzes Peircean continuity, an essential part of any book on Peirce's philosophy of mathematics. A final essay contains an historical examination of the relationship between Peirce and Georg Cantor, and the extent to which Cantor's work influenced Peirce's.

Historical Context. Three of these articles compare Peirce's philosophy directly to 20th century schools of mathematical thought, comparing and contrasting Peirce to structuralism, intuitionism, and realism. Along the way, the reader obtains a thorough introduction to Peircean mathematical metaphysics. Christopher Hookway attempts to characterize Peirce's philosophy as a species of mathematical structuralism. Ahti-Veikko Pietarinen likewise attempts to compare Peirce's diagrammatic pragmaticist philosophy of mathematics to the larger schools of the 20th century, but finds no easy fit. While there are clear similarities between Peirce's approach to mathematics and intuitionism, Hilbert's axiomatic program, quasi-empiricism, and yes, structuralism, Pietarinen finds enough that is unique about the Peircean approach to distinguish it from them all. Particularly interesting is the comparison between Peirce's pragmaticism and intuitionism, both of the general and the Browerian varieties. Lastly, Claudine Tiercelin discusses in detail Peirce's realist-but-not-Platonistic philosophy of mathematics, as an alternative to the epistemological and semantic troubles which face Platonism, without necessitating the adoption of non-realist philosophy of mathematics.

These essays, especially in contraposition to each other, have the overall effect of convincing at least this reader that while Peirce's thinking did in fact have theoretical similarities with various schools of thought, it fits uneasily into any particular school, perhaps providing a substantial alternative to the traditional categories of mathematical metaphysics. Most thought provoking is Tiercelin's argument that a study of Peirce can provide the basis for a non-Platonic realism; much work could be done in this area along Tiercelin's suggested line of inquiry.

Unique in this section is Sun-Joo Shin's article, which is engaging, accessible, and presents its single philosophical point with remarkable clarity. It tackles the long-standing issue of the justification of the status of mathematical truth, focusing on the 'Euclidean triangle' problem: how, precisely, does a person prove a general property of all triangles, through proofs which rely on diagrams of a particular triangle? Shin presents the attempted solutions of John Locke, George Berkeley, and Immanuel Kant, and then employs Peirce's distinction between two different sorts of abstraction to solve the puzzle much more satisfactorily. The article provides an illumination of the nature of Peirce's particular epistemology of mathematics, as well as a clear demonstration of the usefulness of his approach.

Peirce's Mathematical Inquiry. Peirce's mathematical theory is unique, and possibly singular, in that he spends much time investigating the actual process mathematicians undergo when formulating their theorems. Though he does believe the product of mathematics is a collection of necessary truths, the philosophy of pragmaticism demands that the process of creating these truths is thoroughly understood in order to understand the product. Peirce claims the three abilities mathematicians need in the course of their work are imagination, concentration, and generalization. Daniel G. Campos focuses on elucidating Peirce's theory of mathematical imagination, examining the function of imagination in hypothesis-making and mathematical reasoning. Susanna Marietti focuses on another aspect of mathematical creation, namely observation. According to Peirce, mathematical deduction proceeds from hypothetical, idealized diagrams by means of experimentation. While mathematical imagination is necessary for the creation of the hypotheses, as Campos elucidates in the previous article, a peculiar form of *observation* is required to perform these experiments on mathematical diagrams and note the effects. Marietti demonstrates how such a semiotic observation of diagrammatic manipulation preserves the universality and certainty of mathematical knowledge, and the fertility of mathematic investigation.

Elizabeth F. Cooke focuses on the similarity and differences between mathematics and other scientific fields of inquiry. While Peirce sometimes speaks as though mathematics is just like any other scientific field, at other times he wishes to distinguish it based on two features: the necessity of the conclusions, and the uniqueness of mathematical objects of study. The unique nature of the objects of mathematics supposedly changes the methodology, as we do not have the 'check' of the real world available to many other sciences. Cooke argues, however, that mathematical objects are not so distinct as one might suppose, and in fact possess a kind of quasi-secondness, located in the symbols and diagrams necessary to Peirce's mathematical methodology. As such, Cooke finds mathematics to be very similar to the other sciences in terms of methodology, and Peirce's distinction fails.

Continuity. As mentioned above, Peirce viewed understanding continuity as an essential part of understanding the world, and spent much of his life forwarding, rejecting, and refining theories of continuity, attempting to forge a mathematical definition that satisfied his philosophical intuitions. Toward the end of his life, he theorized a 'supermultitudinous collection', the only type of collection which he could consider continuous. Unlike the set of real numbers, for example, supermultitudinous collections are not actual collections, in that they do not have distinct members. Peirce came to believe that, in order to satisfy continuity, every part of a continuum must resemble every other part, except with respect to size; thus, every part of a continuum must itself be continuous. As such, he believed that a continuous line, for example, contained no distinct points, but that we could imagine a continuum forged of points only if there were so many points that they merged and melded into one another. Two of the articles in this section attempt to use modern mathematical theories to give mathematical shape to this rather amorphous supermultitudinous continuity.

Fernando Zalamea's article contains an interesting interpretation of Peirce's supermultitudinous theory of continuity, construing it as a purely modal entity lacking any definite size; he then uses the tools of category theory to 'give a great technical precision to these vague and general initial ideas' (205-6).

Philip Ehrlich presents a much-needed formalization of Peirce's supermultitudinous continuum, using the tools of J. H. Conway's surreal number theory. In particular, Ehrlich shows that there is a remarkable similarity between Peirce's supermultitudinous continuum and a substructure of the surreal number field limited to only its finite and infinitesimal members; this substructure presents a model of what Ehrlich terms the 'Peircean linear continuum'. The essay contains enough background on surreal systems that those unfamiliar with this unique branch of mathematics can still follow the arguments. It also contains arguments separate enough from the details of surreal theory that one can easily grasp Ehrlich's new results without delving into the mathematical details. Thus Ehrlich provides a double service; an introduction to surreal theory, and a formalization of Peirce's supermultitudinous continuum; the reader can benefit from either or both quite independently, but of course the article is richest if one follows it all the way through. Ehrlich's formalization has significant differences from supermultitudinous continuity itself, but the differences are illuminating.

Jérôme Havenel's article, rather than grappling with continuity directly, focuses on a related topic, topology. Though Peirce's work was contemporaneous with important work on the beginnings of topology, he seemed to have been largely ignorant of much of it, and his own work on topology is important in its own right. Havenel surveys Peirce's topological thought, his interactions with other topologists, his own theories on many classic topological puzzles, and he provides a very helpful lexicon of Peirce's terminology on the subject.

'*Peirce's Cantor'*. Matthew Moore's own article is a detailed look at Peirce's reading of Georg Cantor's mathematical works, invaluable for those researching Peirce's theory of continuity in particular. Peirce was highly influenced by Cantor, and his own work overlapped Cantor's in many places. In addition to addressing influence, Moore also addresses the issue of prioricity. Peirce claimed prioricity on the diagonal proof of Cantor's Theorem. Moore shows handily that Peirce's own proof of the theorem was, at the least, written several years after Cantor published his, and thus Peirce's claim to be the first to prove it was clearly false, though Moore argues it is at least possible that Peirce neither plagiarized Cantor's proof nor derived the theorem completely independently, but that, rather, some middle explanation is the correct one, e.g., that Peirce was inspired by Cantor's methodology and thought to derive the theorem he had not yet read. In the final section of this paper, Moore points to the possible development of a pragmaticist metaphysics of sets and membership, along Peircean lines, that could help address some issues raised by current set theoretical approaches.

The collection as a whole is an essential reference for any interested in Peirce's metaphysics and epistemology in general, and mathematical thought in particular, but it also helps to establish Peirce's place as an important figure in the philosophy of mathematics.

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