Penelope Maddy
Defending the Axioms: On the Philosophical Foundations of Set Theory.
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One of the central issues of the philosophy of mathematics is the status of mathematical axioms. Are they simply true? How do we know that? The issue is most pressing with respect to set theory as set theory can either be seen as the foundation or a unifying framework to all of mathematics. Starting from her early paper 'Believing the Axioms', Penelope Maddy contributed in several books and papers to the philosophy of mathematics and set theory. Her position developed and changed over the years, the latest installment is *Defending the Axioms.* The title, however, is misleading. Apart from a few general paragraphs reflecting on the Axioms of Determinacy and Large Cardinal Axioms the book deals deeply with or defends neither specific set theoretic axioms nor set theory. Rather, it defends a broader philosophy of mathematics. This again includes set theory (and often refers in its examples to the practice of set theorists), but could as well be argued with respect to mathematics in its generality.

The book is mostly informal and requires little mathematical or set theoretical knowledge. The difficulty lies not with technicalities but with understanding what Maddy's 'Thin Realism' is supposed to be—and why it should be considered a version of realism at all.

Thin Realism ('TR' for short) is defined as a version of 'Second Philosophy', which is understood as a philosophical stance which does not criticize science in view of (*a priori*) philosophical standards (of method or metaphysics), but sees science as basically correct in the way it is promulgated by scientists. Science includes, for Maddy, mathematics, as she sees mathematics in continuity with the empirical sciences. As there exists an established practice of physics, so there exists an established practice of set theory—and as physics is to be considered as being (roughly) on the right track, so are set theory and mathematics. The stance of 'Second Philosophy' is an epistemological axiom for Maddy. Friends of a 'First Philosophy', who may criticize or re-construct scientific practices—especially general interpretations put by scientists on their theories—to fit in with a broader philosophical world view in metaphysics and epistemology, may resist TR from the very first step.

TR is contrasted with 'Robust Realism', which is what is usually understood by mathematical realism (i.e., mathematical objects exist independently of us, making our theories of them true or false). Maddy claims that TR shares the (Robust) realist's view on mathematical objectivity, but is 'thin' because it bases this attitude on trust of an established scientific practice rather than on a metaphysical world view. The difficulties in understanding the distinction centre on the issue of mathematical objectivity. Where (standard) mathematical realists appeal to mathematical objects and their features, Maddy's TR appeals to 'mathematical depth' in the development of the interconnections of a mathematical field. 'Mathematical depth' increases if theories can be unified and expanded. And supposedly it increases also by providing models for scientific usage. Maddy prefers this 'external' justification of axioms and methods, in contrast to an 'internal' justification within mathematics. She honestly admits that she has not 'given a satisfactory account of mathematical depth—what I've said remains uncomfortably metaphorical' (117, cf. 81); but as honest and humble as this admission is, it leaves the reader wondering about the concept. In particular Maddy's occasional reference to V (the universe of ZFC) as 'the universe that set theory is investigating' (63, cf. 80) does not sound too 'thin' at all, especially since V cannot be an object of set theory itself: the question 'What *is* V?' has been discussed controversially. The whole idea of trusting the set theoretic practice seems also to be rooted in Maddy's conviction that we do not and will not see 'conflicting, equally attractive theories of sets' (63). Non-standard set theorists will demur.

If sets—to take them as paradigm—are nothing over and above what is said in an established scientific practice, this sounds more like constructivism than realism. TR, however, is distinguished from constructivism by Maddy. One reason is that TR appeals to *tertium non datur* (part of the standard practice of logic and set theory), and thus regards, say, the Continuum Hypothesis as either true or false. A second reason is that TR does not simply embrace ZFC as a pragmatically chosen framework, but as 'a body of truths' (70). Later (112), however, Maddy states that applying 'true' and 'exists' in mathematics is an 'idiom' which 'isn't forced upon us'! This again sounds like pure conventionalism: choosing an idiom for practical purposes.

There is another indication that TR is not so realistic. Asking whether our practice may not be wrong after all, TR answers against skepticism that 'sets simply are the sort of things we can find out in these [set theoretic] ways' (75). Now, as a refutation of skepticism this sounds completely constructivist; a realist, in contrast to TR, would say that the 'are' in this reply is open to a realist gap between what *we believe the sets are* and what *they are*. In denying this skeptical possibility, always accepted by ('robust') realism, Maddy moves her TR away from realism. She allows radical skepticism with respect to the senses, but not with respect to our mathematical faculties. Descartes did allow that! There surfaces a deeper issue here, familiar from the debate about Artificial Intelligence, concerning whether there can be a distinction between simulated mathematics and real mathematics; and here—for this reviewer at least—the upshot seems to be that TR is at most a realism about *conceptual* structures.

On the other hand, TR is compared to 'Arealism', which is roughly equivalent to what today is usually called 'fictionalism'. The difficulty here centers on what TR might mean by claiming that sets exist. 'Mathematical depth' seems open also to the 'Arealist'. Although Maddy objects (without reason, on this rare occasion), the development of fiction may be quite constrained, just like the further development of set theory—as can be seen by constraints on sequels and prequels. Maddy comes to the conclusion that both TR and Arealism are justifiable perspectives—'indistinguishable at the level of method' (100)—and sees this as preserving TR as an option even for those with anti-realistic inclinations. If the two really were equally justified, arguments of simplicity and sufficient justification should favor anti-realism, however. TR survives only, it seems, since 'Second Philosophy' cannot employ such (metaphysical) meta-reasoning! TR thrives on an unquestioned trust in the practices of the sciences and appearing 'more natural' (105), i.e. closer to common sense.

Defending the Axioms provides a sketch of another option in the philosophy of mathematics. Mathematical realists (in the ordinary sense) may benefit by trying to argue why they want more than TR. Mathematical anti-realists may get more grist for their mill. However, to see whether TR is leading anywhere, most readers will likely require further clarification.

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