Lloyd Strickland and Harry Lewis, eds. *Leibniz on Binary: The Invention of Computer Arithmetic*. MIT Press 2022. 242 pp. $35.00 USD (Paperback ISBN 978026254434).

Gottfried Wilhelm Leibniz is well known as a philosopher and a mathematician. Leibniz's ideas about necessity and possibility, about a universal language for philosophy and the sciences together with a calculus of concepts to deduce absolute truths, had a substantial influence on the development of philosophy. Leibniz's ideas about infinitesimals and his calculus transformed analysis in mathematics. This book shows that Leibniz made fundamental discoveries in the *theory of computing* that are taken for granted nowadays.

The collection comprises 32 writings, (30 by Leibniz and 2 letters by others), 29 of which have never been published in English. The originals are in Latin, German or French; the translations indicate the sources, alternative transcriptions or translations, deletions and insertions, etc. The precision and thoroughness of Strickland and Lewis in the presentation of the texts is splendid. Strickland and Lewis also provide an introduction that gives context for Leibniz's writings, a brief history of number systems using various bases and an outline of the impact Leibniz's ideas had on the design of some electronic computers. Footnotes accompany the texts; some of them pertain to variations on the text, but most are explanations of the mathematical circumstances. Strickland and Lewis use the supplementary material to painstakingly refute recent plagiarism charges against Leibniz (11-13) and to dispute ‘alternative views’ on the significance on Leibniz's work on binary arithmetic (21). An extensive bibliography and a detailed index conclude the book. Readers might savor looking at Leibniz's manuscripts in the form of 23 reproduced pages.

The *binary progression* (i.e., base-2 arithmetic) is the focal point of the collection, which is necessarily a selection from the immense *Nachlaß* of Leibniz. It seems that Leibniz came to the binary notation around 1674 (28-30), and the same manuscript might contain a hint for his enduring interest in the periods in sum matrices. The piece contains the alternating sequence for π/4, ‘the value of the circle,’ i.e., the area of a circle with unit diameter (61), that Leibniz discovered around the same time. A remarkable feature of 1− 1/3 + 1/5 − 1/7 + 1/9 − 1/11 etc. is that it is an infinite sum of fractions (i.e., rational numbers). In the late 17th century, it was not known whether π was rational; if it were rational then that would have solved positively the *quadrature problem* (i.e., squaring the circle). The irrationality of π was proved by J. H. Lambert in 1761 (45 years after Leibniz's death) and π was proved to be transcendental by F. Lindemann in 1882. The known decimal expansions of π were short (less than 40 digits) and Leibniz noted the lack of formulas to calculate the digits with reference to Ludolph van Ceulen (182). The 6th manuscript (#6) is dated by Leibniz as 15 March 1679, and this piece contains numbers in binary, conversions between binary and decimal, operations on binary (+, −, ·, ÷), two's complement, a method of determining digits in squares and expansions of fractions in binary. Leibniz emphasizes how easy the binary arithmetical operations are, because there are only two symbols; hence, the ‘Pythagorean tables’ are not needed. The simplicity of the operations justifies his claim that they could be carried out by a machine; he sketches one briefly. Leibniz had designed a decimal *calculating machine*, which was presented at the Royal Society in London in 1673. Base-2 would allow a simpler design such as he imagined: balls rolling in channels and dropping into various boxes. He also says (59) that 'But the main question, and the summit of all analysis, is whether any quantity whatsoever is analytical.' The term 'analytic' means calculable (59 fn. 21); hence, for instance, it is a question whether the alternating sequence for π/4 yields a periodic expansion in decimal or in binary. Leibniz worked on the periods more intensively toward the end of the 1690s; each piece from #19 to #27 mentions periods. #27 contains two lemmas, the first of which says (158) that '*a summatrix of a periodic sequence is periodic*,' that is, the sequence of sums of initial segments of a periodic sequence are periodic. The second lemma ensures that addition of columns of periodic digits results in a periodic column.

Leibniz introduced *base-16* (nowadays popularly called hexadecimal) numbers. In #8, he gives a list of numbers in base-16 together with new German names for the new numbers such as '1l' ('*lazehn*'). After this first –musically inspired– notation, Leibniz introduced at least three other notations for digits between '10' and '15' (70 and 130) before settling on 'a,' 'b,' 'c,' 'd,' 'e,' 'f ' (xiv), in 1703. He proclaimed that 'binary progression is for theory, sedecimal for practice' (69), which may explain why he seems not to have carried out investigations of periods in sedecimal. Leibniz thought about how to champion the binary system; indeed, in a letter in 1697, he listed 10 (not 2!) features of the binary that he considers advantageous such as the ease of multiplication and division (111). He also pointed out that assayers have been using weights, which are practically equivalent to the binary system.

Leibniz considered various alternative *notations* for binary (70, 78 and 130); none of those notations would have made it easier to grasp binary numbers. It seems that Leibniz had a hard time to interest his fellow scholars in the binary progression, perhaps, because he did not explain in his communications his motive for his research into periodicity. Thus, a theological interpretation, which particularly pleased one of his employers, a certain Duke Rudolph, was handy to warrant research into binary arithmetic. The analogy between '0' and nothing is hardly new, but the analogy between '1' and unit together with the claim that all numbers are built from 0 by the help of 1 appears to rhyme with creation *ex nihilo* (85). Another selling point for binary presented itself in the form of an interpretation of the hexagrams of Fuxi. Leibniz maintained wide-ranging correspondences including with Jesuits working in China. In his letters (#19, #22, #29), he reported on European scientific and political news, and the invention of binary arithmetic, especially, with a theological analogy attached, was news. Joachim Bouvet responded with a diagram of the 64 hexagrams (six solid or broken lines stacked) and with his view of the hexagrams as binary numbers (#28). It is not completely clear why Bouvet made the suggestion to Leibniz, given that he remained silent about his discovery elsewhere. The connection was questioned already in 1704 by César Case, a Huguenot living in Amsterdam, who was producing calculating machines himself (as well as other scientific equipment) (199-201). Leibniz's reply (#31) to Case is defensive on the interpretation of the hexagrams, however, he wants to provide some intellectually thrilling content. He candidly states (202) that 'the main use of binary arithmetic would be to perfect geometry in relation to the expression of infinitely determined sequences. … If Ludolph [van Ceulen], in giving the value of the circumference 3.141529 etc. had found the way of continuing these digits to infinity by a law of progression, he would have done in whole numbers what I did in fractions, by showing that if the square is 1, the circle is 1− 1/3 + 1/5 −

$1/7 + 1/9 - 1/11$ etc. This determination of the law of progression in whole numbers would be the height of arithmetic applied to geometry' Then he goes on to extol the idempotence of binary multiplication (algebraically, $xx = x$), due to $1 \cdot 1 = 1$ and $0 \cdot 0 = 0$.

This collection will be an enjoyable reading to those who are interested in the philosophy of mathematics. Some pieces give insight into the *fluidity of concepts* in the development of mathematics, such as the notion of transcendental numbers, and how they were still entwined with metaphysical connotations in the 17th century. The manuscripts provide an invaluable peek into how a great mathematician worked and this should delight philosophers who accentuate *mathematical practice*. Writings that Leibniz intended for publication are much more polished and structured than bits and pieces of his work on loose pages. But the latter make it clear that he was developing his ideas about binary arithmetic for years: he created tables of numbers, developed step-by-step methods to calculate, and he also made various little mistakes in calculations in drafts and scribbles. Philosophers and historians of the 17th century may benefit from seeing how a leading thinker of the time saw various political and scientific events, or what the relationship of Leibniz was to various institutions and power holders in Europe.

Finally, philosophers concerned with computation and the invention of computers should start the history of the subject some 200 years earlier than it is customary. Leibniz devised and had constructed more than one *calculating machine* – though they did not work well (or at all), in his lifetime. He *invented the base-2* and *base-16 systems*, the former of which directly impacted Konrad Zuse's design of his computers in the 1930s.

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