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Greg Restall & Shawn Standefer. *Logical Methods*. MIT Press 2023. 284 pp. $40.00 USD (Paperback 9780262544849); $28.99 USD (eBook 9780262372701).

Greg Restall and Shawn Standefer's *Logical Methods* is an advanced textbook on deductive formal logic. Written by two accomplished logicians with extensive teaching experience, the book caters primarily to logic instructors and students already familiar with formal systems. In this review, I will highlight features particularly relevant to educators considering adopting this book for their courses.

The content of a logic textbook and the sequence in which topics are presented are crucial considerations for any instructor. *Logical Methods* focuses on deductive logic, rather than inductive, probabilistic, or informal approaches. While these distinctions are implied in the text, instructors will need to introduce students to the broader differences between formal deductive logic and other forms of reasoning. This textbook covers formal systems such as classical and intuitionistic propositional logic, classical predicate logic, and selected systems of propositional modal logic. Quantified modal logic is addressed briefly in the final chapter. The logical operators include the familiar set: propositional connectives (¬, ∧, ∨, →), quantifiers (∀, ∃), and modal operators (□, ◇).

Logical Methods adopts an innovative structure. Part I (Chapters 1-6) focuses on propositional logic, Part II (Chapters 7-9) covers propositional modal logic, and Part III (Chapters 10-11) addresses predicate logic. The final chapter, Chapter 12, serves as a coda. This arrangement, with modal logic placed between propositional and predicate logic, is unconventional but well-motivated by pedagogical considerations. Another notable innovation is the simultaneous presentation of intuitionistic and classical logics, rather than treating them sequentially.

Two distinguishing features of the book deserve special attention. First, it balances proof-theoretic and model-theoretic approaches. Traditionally, logic texts favor one approach: either starting with semantics (e.g., truth tables for propositional logic or models and possible worlds for predicate and modal logic) and deriving proof systems from that understanding, or beginning with proof theory and later introducing semantics to capture the meanings of the operators as reflected in their behavior in proofs. In contrast, *Logical Methods* gives equal weight to both perspectives, allowing readers to grasp the meaning of logical operators from both syntactic and semantic angles after their informal English interpretations are clarified.

Second, the order of topics within each part varies. In propositional and predicate logic, proof systems are introduced before model theory, but in modal logic, the order is reversed. This change in sequence may require instructors familiar with more traditional textbooks to adjust. The text's equal emphasis on proof and model theory also challenges those with a philosophical preference for one approach over the other.

The following is an outline of the book's content. Chapter 1 introduces key concepts such as arguments, premises, conclusions, and partial orders and trees, the latter being essential for understanding natural deduction proofs. Chapters 2 and 3 present the propositional connectives (negation, conjunction, disjunction, and conditional) along with their introduction and elimination rules, using a Gentzen-style natural deduction system. Notably, this system is initially framed for intuitionistic logic, following Gentzen's original 1934 approach.

Chapter 4, titled 'Facts about Proofs and Provability,' delves into metatheorems and concepts like *normalization* (the transformation of a proof into its normal form, that is, a proof without detours) and the *subformula property* (the restriction of formulas within a proof to subformulas of the premises or the conclusion). It is proved that the intuitionistic natural deduction system possesses both features. This chapter is considerably more challenging than its predecessors, and although it is marked with a "Warning Label," it could have benefited from a more comprehensive introduction to guide the reader. The chapter ends by elucidating the difference between intuitionistic and classical logic, demonstrating that the law of excluded middle (any sentence or its negation must be true) does not hold in intuitionistic logic. The text clearly and informatively explains how classical logic can be obtained by adding the double negation elimination rule (from ¬¬A, one can obtain A) to intuitionistic logic.

Chapter 5 introduces the semantics of propositional logic, focusing on Boolean truth-value assignments, which correspond to classical logic. In Chapter 6, the soundness and completeness of classical logic are proved. It is then shown that while intuitionistic logic is *sound* with respect to Boolean models (every valid argument in intuitionistic natural deduction is also valid in Boolean models), it is not *complete* with respect to Boolean models. That is, there are valid arguments in Boolean valuations that are not provable in intuitionistic natural deduction. In an appendix, Heyting algebras are introduced as a replacement for Boolean valuations, providing a semantic framework with which intuitionistic natural deduction is complete.

Part II deals with modal logic. Chapter 7 begins with a brilliant introduction that connects propositional and modal logic, equating ‘true on all valuations’ with *necessity* and ‘true on some valuation’ with *possibility*. The first step in modal logic relies on Leibniz’s definition: a proposition is necessary if and only if it is true in every possible world, and possible if it is true in some possible world. This suffices to construct a semantic framework and introduce the first system of modal logic, S5. Later in the chapter, the notion of the accessibility relation is introduced, indicating which worlds are possible from the perspective of a given world. This paves the way for explaining Kripke models, which serve as the foundation for the system S4, a weaker system than S5. In an appendix, intuitionistic Kripke models are defined as another alternative (in addition to Heyting algebras) to accommodate intuitionistic logic.

Chapter 8 is devoted to “actuality models,” where a particular possible world should be designated as the actual one, and ‘double-indexed models,’ where no single world is forced to be actual, and any possible world may be designated as actual. In Chapter 9, a natural deduction proof system is characterized for S4 and S5, and features such as soundness and completeness are briefly addressed.

Part III, on predicate logic, begins in Chapter 10 by defining the language and syntax of predicate logic, including terms, predicates, and quantifiers, and illustrating how well-formed formulas are constructed and how open and closed formulas are distinguished. The chapter introduces Gentzen-style natural deduction for classical first-order predicate logic (CQ) and briefly touches on quantified intuitionistic logic (IQ), showing how it can be obtained either by dropping double negation elimination (DNE) from CQ or by adding quantifier rules to propositional intuitionistic logic. Chapter 11, on model theory, introduces interpretations for names and predicates before tackling variables and quantifiers. It also discusses counterexamples (models in which premises are true and the conclusion false) and proves the soundness and completeness of classical first-order logic. In the last section, “The Power and Limits of Predicate Logic,” the expressive power of predicate logic and its role in enhancing the rigor and precision of mathematics are explored.

The final chapter, “Coda,” includes a brief discussion of quantified modal logic and a helpful “Suggestions for Further Reading” section, explaining the areas of logic covered in the book, as well as those not covered, with suggestions for further exploration.

Although *Logical Methods* is primarily a logic textbook, it occasionally touches on philosophical issues; two sections stand out in this regard. Section 6.3 explores whether proof theory or model theory better captures the nature of logical operators, contrasting representationalism, which argues that the meanings of logical operators are best manifested in their model theory, with inferentialism, which claims that the rules of inference best characterize their meanings. Section 7.3 is devoted to strict implication (when A necessarily implies B), the ambiguity of necessity (the difference between 'A necessarily implies B' and 'A implies that B is necessary'), and the conception of a proposition as the set of worlds in which the sentence is true.

The pedagogical tools offered by this book are noteworthy: (a) each chapter begins with a clear introduction outlining its purpose and its relation to the preceding chapters; (b) chapters conclude with exercises, divided into basic and challenge questions, which are valuable for both students and instructors; and (c) the book includes a glossary, a symbol index, and a subject index, though the latter is missing some key terms such as quantification, meta-theory, and two-dimensionality.

While *Logical Methods* claims to require no prior knowledge of logic, its depth and complexity make it more suitable for advanced students or those with backgrounds in related fields such as computer science or mathematics. It is unlikely to be appropriate for first-year undergraduates.

In sum, *Logical Methods* is an innovative and rigorous textbook that will challenge both instructors and students. Its novel structure and balanced approach to proof and model theory distinguish it from more conventional introductory logic texts. While not without its difficulties, it offers substantial rewards for those who invest the effort to navigate its complexities.

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