Mark Colyvan
An Introduction to the Philosophy of Mathematics.
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Colyvan's textbook is an absorbing introduction to key topics in the philosophy of mathematics since the pioneering work of Paul Benacerraf. It focuses mainly on the issue of mathematical realism – the question of whether or not mathematical objects exist – whilst offering brief forays into the history of mathematics and into representation and notation. The book is beautifully written, exemplifying the key features that a textbook in philosophy ought to have: it is clear, lively and enjoyable to read. The book also uses a light touch with regard to technical material. Mathematical results, where included, are explained in detail and in such a way that a student with little or no mathematical background can follow the discussion. Colyvan's text is therefore appropriate for implementation at an undergraduate level.

The book's usefulness is not, however, limited to its uses as a primary source in undergraduate course work. It can also be used as the basis for designing graduate level courses, at both a masters and a PhD level. One of the central achievements of Colyvan's textbook is the way it seamlessly weaves together introductory material on the debate over mathematical realism with state of the art research on mathematical explanation, the applicability of mathematics in science and mathematical notation. The book would therefore serve as a useful touchstone for a graduate student writing a research thesis on one of these interesting topics. If nothing else, the extensive bibliography provides a systematic road-map of the latest work in this area.

The book is divided into eight chapters and an epilogue. At the end of each chapter is a list of discussion points and further readings. The discussion points in particular are incredibly useful for designing courses in this area, doing double duty as topics for class discussion and as schema for formulating essay and exam questions. The readings provide useful sources for research essays and for keen students who want to dig a bit deeper into any of the topics that the book covers.

Chapter One of the book provides an introduction to the relationship between mathematics and philosophy, before moving very quickly through a bit of the history of mathematics and of the philosophy of mathematics. The central topics that the book aims to explore are then introduced via Benacerraf's two big papers in this area: 'What is Mathematical Truth?' and 'What Numbers Could Not Be'. For some, the short shrift given to the 'big isms' – as Colyvan calls them – logicism, formalism and intuitionism, will be unsatisfying. However, it must be kept in mind that Colyvan offers *an* introduction to the philosophy of mathematics, not *the introduction to end all introductions*. Accordingly, the book is focused on issues of realism, which – in contemporary debates at least – are largely orthogonal to the big isms charted in Chapter One.

Chapter Two is a brief introduction to two issues in the foundations of mathematics, which possess philosophical significance. These are the Löwenheim Skolem theorem and its connection to Cantor's set theoretic hierarchy, and Gödel's incompleteness theorems. Both theorems are briefly introduced and explained, and then used to motivate the issue of realism in the philosophy of mathematics.

The question of whether or not mathematical objects exist is taken up in earnest throughout Chapters 3–6. Chapter Three differentiates between various versions of realism about mathematics, paying careful attention to the distinction between realism about mathematical *truths* and realism about mathematical *objects*. Colyvan focuses on the second form of realism, noting only that 'it seems a very quick path from objective truth to objects' (37). He goes on to lay out the central argument in favour of realism about mathematical objects: the (in)famous indispensability argument.

Chapter Four considers the chief opponent of the mathematical realist, the fictionalist. It provides a brief overview of Field's nominalisation program – which Colyvan calls the hard road to nominalism – before canvassing alternative lines of resistance, so-called 'easy roads' to nominalism. Easy road strategies are easy because they do not attempt to undertake the daunting task of stripping mathematics out of our best scientific theories. Instead, they attempt to undercut the indispensability argument in other, arguably more subtle ways. Colyvan's discussion in this chapter closely follows his 2010 paper 'There is No Easy Road to Nominalism' in *Mind*.

Following the discussion of fictionalism in Chapter Four, Chapter Five introduces the recent 'explanatory turn' in the debate over mathematical realism. The explanatory turn involves the reformulation of indispensability arguments to focus on explanation. Colyvan outlines some examples of extra-mathematical explanation – the explanation of physical facts by facts about mathematics – and briefly considers the topic of intra-mathematical explanation – the explanation of mathematical facts by one another. The chapter also provides an introduction to recent theories of explanation, as a way to understand the explanatory power of mathematics.

Chapter Six moves on from the indispensability argument to another topic of great interest: the effectiveness of mathematics in science. The usefulness of mathematics within science has been thought by some mathematicians to be mysterious. Colyvan shows how to develop an account of applied mathematics that goes some way toward addressing the charge of mystery. This is his 'mapping' account which holds, roughly, that mathematics gets applied in science via the combination of functional mapping into and out of mathematics combined with certain inferential connections between fragments of mathematical theory.

The final two chapters feel like 'bonus chapters'; they do not gel quite as well with the narrative arc of the rest of the book. Still, they add to the richness of the text by raising two rather controversial issues. Chapter Seven broaches the topic of inconsistent mathematics, and the idea of applying such mathematical theories in science. This takes the book into the domain of non-classical logics (namely, paraconsistency). Chapter Eight shifts gears again to focus on mathematical notation, and the idea that progress in mathematics can sometimes be chalked up to revolutions in mathematical formalism. One striking example of this, discussed by Colyvan, is the shift from the Roman numerals to the Arabic numerals. Colyvan contends that the Arabic numerals suggest a range of number theoretic results that are not suggested by the Roman numerals. For instance, the Arabic numerals have recursion built in; the Roman numerals do not.

The Epilogue is a list of 'desert island theorems'. These are the theorems that Colyvan believes every philosopher – and thus every budding philosopher, undergraduate or graduate – should know. Many of these theorems have found their way into philosophy, and where they have Colyvan notes their impact and relevance.

Despite its aforementioned focus on realism, *An Introduction to the Philosophy of Mathematics* actually covers a surprisingly wide range of topics. Far from being a flaw, this increases the utility of the book with regard to course design. There are, at least, two courses that could be taught from the book. One course would centre on issues to do with realism in the philosophy of mathematics and would be well-suited to upper level undergraduates. Having taught a course like this using Colyvan's text book at the University of Sydney, I can attest to the book's practical utility. What I discovered when teaching from the book is that its focus on recent research in this area is stimulating for students, who become excited by the opportunity to try and say something new about a topic of ongoing debate. This was pedagogically invaluable and made for a much more enjoyable course to teach, one in which students were extremely engaged.

A second course that might be taught from the book would focus more on issues surrounding realism, including mathematical history and application as well as notation and explanation (of the intra-mathematical kind). Such a course could do well at both an advanced undergraduate or a graduate level. Colyvan's text book could also be used as a supplementary text for a course on the philosophy of science. Much of the debate over realism in the philosophy of mathematics bears deep connections to debates over scientific realism more generally. Accordingly, the indispensability argument can be used as a kind of toy model for exploring the issue of ontological commitment in scientific contexts. Chapters Three through Six would be especially useful for this purpose.

In sum, this is an excellent textbook, and a fruitful one at that. It takes students to the cutting edge of research in this area and allows them to look into the abyss, which has a stimulating effect on pedagogy. In the end, Colyvan teaches us all a lesson about writing textbooks. Taking a textbook to the edge of uncharted terrain is a very good thing as it thereby provides a basis for some pretty exciting teaching.

Sam Baron
University of Western Australia