

Lloyd Humberstone

The Connectives.

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Lloyd Humberstone's *The Connectives* is a gigantic book about propositional logic. The titular connectives are 'and', 'or', 'if', and 'not'; each is studied extensively using the tools of modern mathematical logic. The book is nominally concerned with the "semantics and pragmatics of natural language," with formal discussion couched in algebraic techniques (Galois connections appear on page 3). What is most striking about the book is its sheer size. For example, the chapter about 'and' – the first connective to receive full attention – does not appear until page 631, and is itself over 130 pages long. The chapter on 'If,' which arrives on page 925, is longer. Indeed, *The Connectives* is nearly 1,500 pages of densely packed rumination over, well, the propositional connectives.

It should be relatively obvious to prospective readers whether or not this is a book for you.

Because of its magnitude, *The Connectives* is also very hard to review – should I dedicate one word to each page? There is no synopsis or substitute for pouring over this tome. I will keep my remarks impressionistic, intended for a general philosophical audience.

There are four substantial chapters of general logical theory before getting to the connectives. In chapter 1 we examine the core concepts of truth, consequence, rules and proof. Chapter 2 covers many-valued and modal logics, with algebraic semantics; Chapter 3 is about properties of connectives such as truth-functionality and extensionality. The last preliminary chapter, Chapter 4, is concerned mainly with the existence and uniqueness of the connectives. This phase of the book could have been a monograph on its own.

The central chapters (5–8) on the connectives follow a general pattern. First some natural language behaviors of, e.g., disjunction are introduced, with some nice examples. For instance, a close reading of *Anna Karenina* shows that the novel may begin on a Monday or a Tuesday – prompting one critic to comment that "nothing turns on this inconsistency." Inconsistency? Over Monday *or* Tuesday? Perhaps: according to one account, there are contexts in which 'or' can mark a wide-scope *conjunction*, just one of its many roles. (Humberstone is as meticulous as he is erudite: he tells us that the example comes from Banfield, quoting Heinz, who was quoting Savile (in discussion) – and it turns out that Banfield misquotes Heinz, who wrote "a Monday *and* a Tuesday" after all.) Or – using 'or' in the wide-scope sense – in the chapter on negation, Humberstone points to Cotard's delusion, a psychological syndrome in which the patient believes herself not to exist, contra the Cartesian *cogito*. This condition is known in French as *délire de négation*.

Once the natural language examples are set aside, the chapters then pinwheel around the symbolic world of logical theory – quantum, minimal, relevant, intuitionistic and modal logics, semantic and syntactic approaches, commas on the left and *falsum* on the right. Scholars are all

given copious credit, both famous and lesser-known lights: just from the “P”s, we meet Paris, Pierce, Popper, Porte, Prawitz, Priest, Prinz, Prior, and Putnam, among many others. The bibliography alone is 98 pages long. *The Connectives* is full to brimming with commentary, criticism, suggestions for further reading, wry insights, the occasional pun – *h’ORs d’oeuvres*, anyone? – all drawing on an apparently bottomless well of first-class logical results.

The book is not meant to be read linearly. The prologue Chapter 0 is a long primer on the algebra needed for the rest of the book, opening with ‘Skim or skip this chapter’. There are copious cross references throughout, encouraging the reader to jump in, skip around, and follow a path of in-book citations until a given concept is clear. This is a very useful feature, if a bit disorienting in a book so big. (When you are feeling lost, there can be such a thing as too much signposting.) Humberstone spends so much effort noticing how topics relate to each other, he sometimes doesn’t quite get around to introducing or motivating some of them. We simply begin, in medias res. Again, Humberstone seems to take it as relatively obvious to the reader whether or not this is a book for you.

In the final chapter, Chapter 9, two very abstract and rather beautiful properties of connectives are discussed. A class of formula D is *universally representative* when every formula of the language is synonymous with some formula in D . So conjunction is universally representative in classical logic, because everything has some conjunctive equivalent. This allows a very high-level view of logics, and it prompts philosophical questions, like whether there are any genuinely disjunctive propositions. Then the class D is *special* iff, roughly, a logic treats members of D differently than other formulas of the language (the exact definition is on page 1302). Often this class D is carved out by a particular connective, in which case the connective is *special* in the logic. The necessity operator is special in normal modal logic. These concepts meet at the result that being special is dual to being universal: No connective universally representable in a logic is special in that logic.

And not long after that observation, *The Connectives* ends, as elliptically as it began. At which point, if not well before, one wants to ask: what is this book *about*? There is no single thesis being advanced, no ‘main argument’ that I could discern. There is a coherent viewpoint. Logic for Humberstone is about a very general notion of consequence. Specific logics are obtained by getting more specific about what notion of consequence is at issue. Logics are generally individuated by what sets of sequents they contain. A consequence relation may go from sets to sets (multiple premise, multiple conclusion), or sets to formula (single conclusion), or even from sets to *no* conclusions. There is emphasis on formulas, contextualized against a background theory of combinators and multi-sets. Humberstone almost exclusively ‘shows’ rather than ‘tells’. If not a thesis, then the book puts forward a distinctive *flavor* of philosophical logic.

At risk of abusing a cliché, *The Connectives* is a *tour de force*, an act of massive cognitive self-expression. I know of no other logic monograph like it. Although it is comprehensive, it is not a systematic (i.e., linear) development; it is not a textbook or a handbook. It could serve well as the last text an advanced reading group will ever need. Nearly every single LaTeX symbol is used. Humberstone offers us an extended, intense and deep meditation on logic, and by the end one suspects he could go on for another 1,500 pages without any trouble. He has, after all, not

even *begun* on quantifiers.

It is easy to get lost in this book. Do.

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